International Journal of Engineering, Science and Mathematics
Vol. 11 Issue 4, April 2022,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

DIFFRACTION OF SH-WAVE BY A GRIFFITH CRACKIN AN INFINITELY LONG ISOTROPIC ELASTIC STRIP

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ABSTRACT

The problem of incident SH-wave response of a Griffith Crack in an infinitely long isotropic elastic strip is analyzed. Fourier transform is used to reduce the mixed boundary value problem to the Fredholm integral equation of second kind. The expressions of elastodynamicstress intensity factor (SIF) at the tip of the crack, shear stress away from the crack and the crack opening displacementare obtained.Finally the integral equation is solved numerically and all the quantities are evaluated for different geometry parameters and shown by means of graphs.

Keywords **:***SH-wave, Griffith Crack, Isotropic, Elastic strip, Mixed boundary value problem, SIF.*

2010 Mathematics Subject Classification: Mechanics of deformable solids: code 180658

1.INTRODUCTION

The study of interaction of cracks and inclusions in elastic medium has been of great interest since long back. The study of elastodynamic response of cracks or inclusions is now much important in view of their applications in geophysics and seismology. Many researchers studied the problems involving one or more cracks in an infinite homogeneous elastic medium. Loeber and Sih[8] and Mal[9] has studied the problem of diffraction of elastic waves by a Griffith crack in an infinite medium. Srivastava et.al[17] and Bostom[1] considered interface crack. Problem involving finite crack perpendicular to the surface of infinitely long elastic strip has been studied by Chen[2] for impact load and Srivastava et.al.[16] for incident wave. Pramanicket.al[14] studied the problem of edge crack in an elastic semi-infinite medium.Matysiak and Pauk[10] studied the edge crack in an elastic layer resting on Winkler Foundation. Edge crack under normal loading studied by Das et.al.[3,4] in the surface of an orthotropic strip bonded to an orthotropic half plane and in the surface of an orthotropic strip of finite thickness bonded to another orthotropic finite strip. Stress intensity factor for two parallel interface cracks has been obtained by Itou[6].

Sinharoy[15]studied the elastostatic problem of an infinite row of parallel cracks. Monfaredet.al[11] obtained the stress intensity factors of multiple cracks in an orthotropic strip with FGM coating. Diffraction of P-waves by an edge crack problem within aninfinite strip have been discussed by Munshi and Mandal [12] and Nandi et al. [13].

In the present paper the diffraction of time harmonic SH-wave by a crack in an infinitely long isotropic elastic strip has been studied. Applying Fourier transform, the mixed boundary value problem has been converted to the solution of dual integral equations which are finally reduced to a Fadedholm integral equation of second kind. The expressions of stress intensity factor at the tip of the crack, shear stress away from the crack and crack opening displacement are obtained. Finally the integral equation is solved numerically and all the quantities are evaluated for different geometry parameters and shown by means of graphs.

2.FORMULATION OF THE PROBLEM

We consider the problem of diffraction of SH-wave by a Griffith crack in an infinitely long isotropic elastic strip of width $2h_1$. The crack is located in the region$-a \leq x_1 \leq a$, $-\infty < y_1 < \infty$, $z_1 = 0$. Normalizing all the lengths with respect to $'a'$and putting $\frac{x_1}{a} = x$, $\frac{y_1}{a} = y$, $\frac{z_1}{a} = z$, $\frac{h_1}{a} = h$, it is found that the location of the crack is $-1 \leq x \leq 1, -\infty < y < \infty, z = 0$(Fig-1) referred to Cartesian co-ordinate system (x, y, z). Let a normally incident time harmonic anti plane shear wave travels in the positive direction of z axis. The oscillatory term$e^{-i\omega t}$ which is common to all field variables, is omitted in the following formulation. Also we assume that the two faces of the crack do not come in contact during vibration.

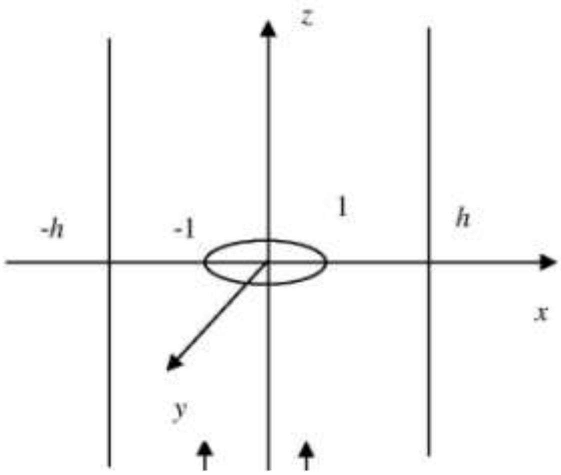

Fig.- 1: Geometry of the crack

Thus the problem is to find the stress distribution near the crack tips subject to the following boundary conditions:

$\sigma_{yz}(x,0) = -q_0|x| \leq 1$ …………….

(1)

$u_y(x,0) = 01 \leq |x| \leq h$ ……………..(2)

whereq_0 is a constant.

The edges of the strip are stress free, so that

$\sigma_{xy}(h,z) = \sigma_{xy}(-h,z) = 0$

…………… (3)

In Cartesian co-ordinate system the displacement vector has the form$(0, u_y, 0)$.

Thus the problem of determining the stress distribution reduces to that of obtaining the solution of the displacement equation

$\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial z^2} + k_2^2 u_y = 0$ ………………….(4)

where$k_2^2 = \frac{\rho\omega^2}{\mu}$ …………………. (5)

ρ is the density of the material and μ is the Lame's constant.

The solution of equation (4) is given by

$u_y(x,z) = 2\int_0^\infty A(\xi)e^{-\beta z}\ cos\xi x d\xi + \int_0^\infty B(\zeta)\cosh(\beta_1 x)\, sin\zeta z d\zeta$

………………… (6)

where $\beta^2 = \xi^2 - k_2^2\ and\ \beta_1^2 = \zeta^2 - k_2^2$

………………… (7)

and $A(\xi)$ and $B(\zeta)$ are the unknown functions.

Now the stress components$\sigma_{xy}(x,z)$and$\sigma_{yz}(x,z)$can be expressed as

$$\sigma_{xy}(x,z) = \mu\frac{\partial u_y}{\partial x}$$

$=-2\mu\int_0^\infty \xi A(\xi)e^{-\beta z} sin\xi x d\xi + \mu\int_0^\infty \beta_1 B(\zeta) sinh(\beta_1 x) sin\zeta z d\zeta$

…………………… (8)

$$\sigma_{yz}(x,z) = \mu\frac{\partial u_y}{\partial z}$$

$=-2\mu\int_0^\infty \beta A(\xi)e^{-\beta z} cos\xi x d\xi + \mu\int_0^\infty \zeta B(\zeta) cosh(\beta_1 x) cos\zeta z d\zeta$ ………………… (9)

3. SOLUTION OF THE PROBLEM

The boundary conditions (1) and (2) yields the following dual integral equations :

$2\int_0^\infty \beta A(\xi)cos\xi x d\xi - \int_0^\infty \zeta B(\zeta)cosh(\beta_1 x)d\zeta = \frac{q_0}{\mu}, \quad |x| < 1$

…………………… (10)

$\int_0^\infty A(\xi)cos\xi x d\xi = 0\ 1 \leq |x| \leq h$ ………………….. (11)

Now from boundary conditions (3) we get the following relation between the unknowns$A(\xi)$ and $B(\zeta)$:

$$\int_0^\infty \beta_1 B(\zeta)sin(\beta_1 h)sin\zeta z d\zeta = 2\int_0^\infty \xi A(\xi)e^{-\beta z}sin(\xi h)d\xi$$

Using Fourier sine inversion we get

$\beta_1 B(\zeta)\sin h(\beta_1 h) = \frac{4}{\pi}\int_0^\infty \frac{\zeta\xi A(\xi)sin(\xi h)}{\beta^2+\zeta^2}d\,\xi$

………………….. (12)

Substituting the values of $B(\zeta)$ from (12) into (10) and (11) we get the following dual integral equations

$\int_0^\infty \xi[1 + H(\xi)]A(\xi)cos\xi x d\xi = Q(x), \qquad |x| < 1$

……………………….(13)

$\int_0^\infty A(\xi)cos\xi x d\xi = 0\ 1 \leq |x| \leq h$ ……………………….(14)

where$Q(x) = \frac{q_0}{2\mu} + \frac{2}{\pi}\int_0^\infty \frac{\zeta^2 cos h(\beta_1 x)}{\beta_1 sin h(\beta_1 h)}\int_0^\infty \frac{\xi A(\xi)sin(\xi h)}{\beta^2+\zeta^2}\,d\xi d\zeta$

……………………… (15)

$H(\xi) = \xi^{-1}\left[\sqrt{\xi^2 - k_2^2} - \xi\right]$

……………………… (16)

and$H(\xi) \to 0\ as\ \xi \to \infty$

Let us consider the solution of the integral equations (13) and (14) in the form

$A(\xi) = q_0(2\mu)^{-1}\int_0^1 tg(t)J_0(\xi t)dt$ ……………………… (17)

So that equation (14) is automatically satisfied and equation (13) can be written as

$\int_0^1 tg(t)\int_0^\infty \xi[1 + H(\xi)]J_0(\xi t)cos\xi x d\xi dt = \frac{2\mu}{q_0}Q(x)$ ……………………… (18)

where $J_0()$ is the Bessel function of first kind of order zero.

Now using Abel's transform in equation (18) we obtain the following Fredholm integral equation of second kind :

$g(t) + \int_0^1 ug(u)L_1(u,t)du = R(t)$

……………………… (19)

where$R(t) = \frac{4\mu}{\pi q_0}\int_0^t (t^2 - x^2)^{-\frac{1}{2}}Q(x)dx$ ……………………… (20)

and$L_1(t,u) = \int_0^\infty \xi H(\xi) J_0(\xi t) J_0(\xi u) d\xi$ ……………………… (21)

Next substituting the value of $A(\xi)$ and (17) in (20) and using the results (Gredshteyn and Ryzhik) [7]

$$\int_0^t (t^2 - x^2)^{-\frac{1}{2}} cosh(\xi x) dx = \frac{\pi}{2} I_0(\xi t)$$

$$\int_0^\infty \frac{x\, sin(xy) J_0(ax)}{(x^2 + b^2)} dx = \frac{\pi}{2} e^{-by} I_0(ab)$$

we get$R(t) = \frac{4\mu}{\pi q_0} \int_0^t (t^2 - x^2)^{-\frac{1}{2}} Q(x) dx$

$= 1 - \int_0^1 u g(u) L_2(u,t) du$

……………………… (22)

where$L_2(u,t) = -\frac{2}{\pi} \int_0^\infty I_0(\beta_1 t) cosech(\beta_1 h) K(\zeta, u) d\zeta$ ………………………

(23)

and$K(\zeta, u) = \int_0^\infty \frac{\zeta^2 \xi sin\,(\xi h) J_0(\xi u)}{\beta_1 (\beta^2 + \zeta^2)} d\xi$

$=\begin{cases} \pi \zeta^2 (2\beta_1)^{-1} e^{-h\beta_1} I_0(\beta_1 u), & \zeta > k_2 \\ -i\pi \zeta^2 \left(2\beta_1{}'\right)^{-1} e^{-ih\beta_1{}'} J_0(\beta_1{}' u), & \zeta > k_2 \end{cases}$ ………………………

(24)

$\left(\beta_1{}'\right)^2 = k_2^2 - \zeta^2 and\ \beta, \beta_1$ are given by (7)

Now substituting (22) in (19) we finally obtain the following Fredholm integral equation of second kind for the determination of the unknown $g(t)$:

$g(t) + \int_0^1 u[L_1(u,t) + L_2(u,t)] g(u) du = 1$……………………… (25)

where $L_1(u,t)$and $L_2(u,t)$ are given by (21) and (23).

It is to be noted that the Kernal$L_1(u,t)$ represented by the semi-infinite integral given by equation (21) has slow rate of convergence. It has no poles. It has only branch point at the points$\xi = \pm k_2$. In order to make the numerical integration easier, the semi-infinite integral has therefore been converted to finite integrals by using simple contour integration technique and is given by

$L_1(u,t) = -ik_2^2 \int_0^1 \sqrt{1 - \xi^2} J_0\,(k_2 \xi t) H_0^{(1)}(k_2 \xi u) d\xi\ u > t$

………………………(26)

The integrand in (23) can be written as

$$L_2(u,t) = -\frac{2}{\pi}\left[\int_0^{k_2} + \int_{k_2}^{\infty}\right] I_0(\beta_1 t) cosech(\beta_1 h) K(\zeta,u) d\zeta$$

In $\int_0^{k_2}$ putting $\zeta^2 = k_2^2(1-y^2)$ and in $\int_{k_2}^{\infty}$ putting $\zeta^2 = k_2^2(1+y^2)$we get

$$L_2(u,t) = -k_2^2\left[\int_0^{\infty} \sqrt{1+y^2} e^{-k_2 y h} I_0(k_2 y u) I_0(k_2 y t) cosech(k_2 y h) dy\right.$$

$$-\int_0^1 \sqrt{1-y^2} cot(k_2 h y) J_0(k_2 y t) J_0(k_2 y u) dy$$

$$\left. +i\int_0^1 \sqrt{1-y^2} J_0(k_2 y t) J_0(k_2 y u) dy\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

4. QUANTITIES OF PHYSICAL INTEREST

Stress field outside the crack: The stress $\sigma_{yz}(x,z)$is significant. The expression of shear stress $\sigma_{yz}(x,z)$can be obtained from equation (9) and is given by

$$\sigma_{yz}(x,z) = -2\mu \int_0^{\infty} \beta A(\xi) e^{-\beta z} \cos(\xi x) d\xi + \mu \int_0^{\infty} \zeta B(\zeta) cosh(\beta_1 x) \cos(\zeta z) d\zeta \quad x > 1$$

$$= -q_0 \int_0^1 t g(t) \int_0^{\infty} \beta J_0(\xi t) e^{-\beta z} \cos(\xi x)\, d\xi dt$$

$$+q_0 \int_0^1 t g(t) \int_0^{\infty} \frac{\zeta^2 e^{-\beta_1 h}}{\beta_1} cosech(\beta_1 h) I_0(\beta_1 t) cosh(\beta_1 x) \cos(\zeta z) d\zeta dt \quad x > 1$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

Stress Intensity Factor : The stress $\sigma_{yz}(x,z)$in the plane z=0 in the neighbourhood of the crack can be found from equation (9) and is given by

$$\sigma_{yz}(x,0) = -2\mu \int_0^{\infty} \beta A(\xi) \cos(\xi x) d\xi + \mu \int_0^{\infty} \zeta B(\zeta) cosh(\beta_1 x) d\zeta \quad x > 1$$

$$= q_0 \frac{x}{\sqrt{x^2-1}} g(1) + 0(1) \qquad x > 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Defining stress intensity factor $K$by

$$K = \lim_{x\to 1^+}\left|\frac{\sqrt{x-1}\,\sigma_{yz}(x,0)}{q_0}\right|$$

we get$K = \left|\frac{g(1)}{\sqrt{2}}\right|$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$

Crack opening Displacement: Next, the displacement component $u_y(x,z)$ on the crack surface can be obtained from (6) as

$$u_y(x.0) = 2\int_0^\infty A(\xi)\cos\xi x d\xi$$

$$= \frac{q_0}{\mu}\int_x^1 \frac{tg(t)}{\sqrt{t^2-x^2}}dt \ , \qquad |x|<1$$

$$= \frac{q_0}{\mu}\left[\sqrt{(1-x^2)}g(1) - \int_x^1 \sqrt{t^2-x^2}\,g'(t)dt\right] |x|<1 \qquad \text{.........................(31)}$$

5. NUMERICAL SOLUTION AND DISCUSSION

Using the method of Fox &Goodwin[5] the integral equation (25) is solved for different values of the strip width *h* (*h*=1.5, 2.5, 3.5) and varying frequency k_2 in the range $0.1 \le k_2 \le 6.0$. The infinite integrals $L_1(u,t)$ and $L_2(u,t)$ were evaluated by using 32-point Gaussian quadrature formula then integral in equation (25) was evaluated by a Sympson's one third quadrature formula involving values of the desired function $g(t)$ at points inside the specific range of integration and then converted to a set of simultaneous linear algebraic equations. The solution of the set of linear algebraic equations gives a first approximation to the required pivot values of $g(t)$ which has been improved by the use of difference correction technique. Using these values of $g(t)$ dynamic stress-intensity factor(SIF) has been calculated and plotted against dimensionless frequency k_2 in Fig.-2 for different values of *h*(*h*=1.5, 2.5, 3.5).

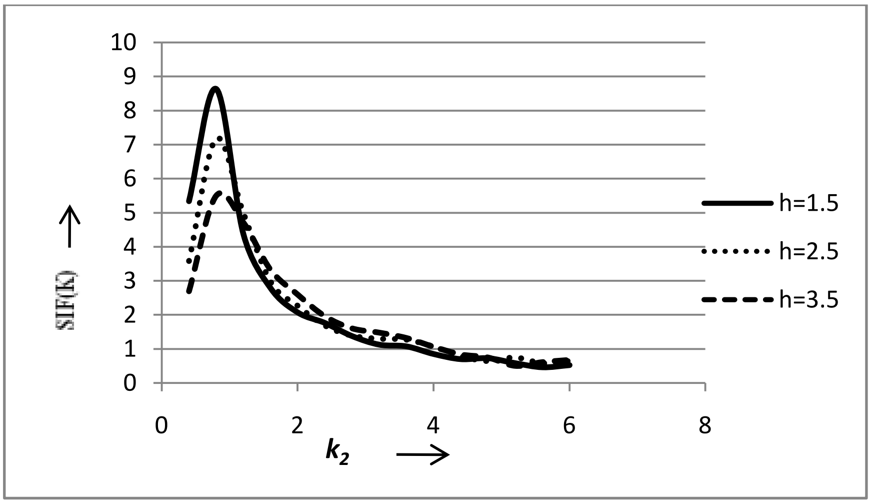

Fig.-2 Stress intensity factor *K* versus dimensionless frequency k_2

In Fig.-2 it is observed that SIF (K) first increases with k_2 reaches a maximum value then decreases with the increase in the values of k_2. It is also observed from the figure that the maximum value of SIF(K) decreases as the width *h* of the strip increases.

In Fig.-3 and Fig.-4 dimensionless Crack opening displacements (COD) are plotted against x $(-1 \leq x \leq 1)$ for different values of k_2 . It is observed that the graph is symmetric about x=0. Also it is to be noted that COD achieves its maximum value at x=0 and gradually increases (Fig.-3) for higher values of k_2 upto a certain value of $k_2$then starts decreasing(Fig.-4).

Next the stressfield at distant points from the crack has been computed numerically from equation (28) and presented by means of 3-dimensional graphs (Fig.-5 and Fig.-6) for various values of h(1.5,3.5) and for a fixed value k_2=0.7. From the plot it is observed that the stress field

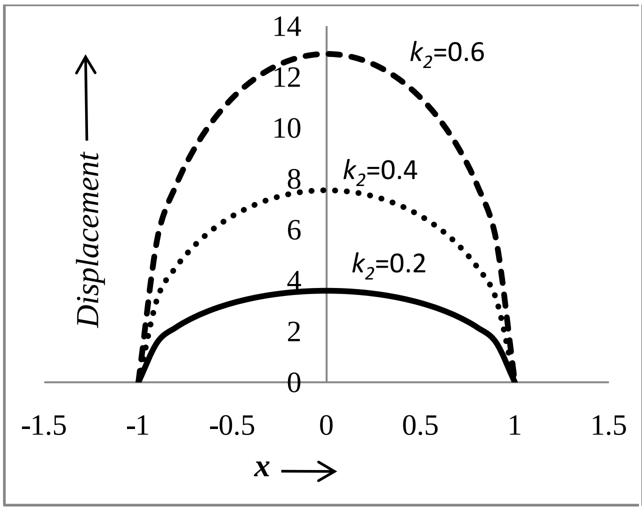

Fig.-3 Crack Opening Displacement(CODversus *x*

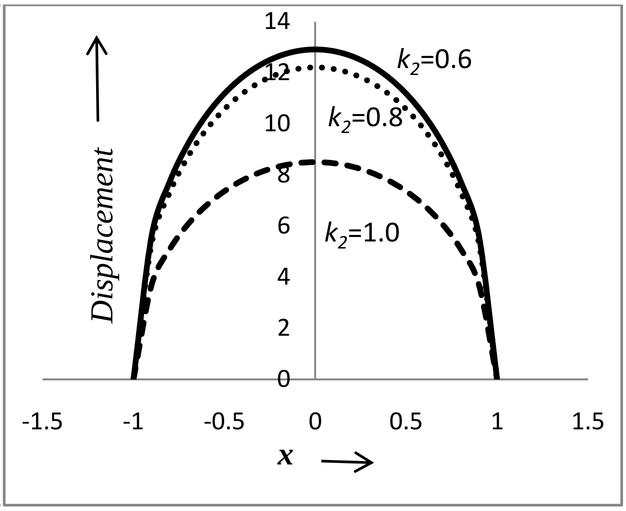

Fig.-4 Crack Opening Displacement(COD) versus *x*

shows wave like nature and very disruptive near the cracktip and the sharpness decreases as x increases. It is also seen that stress decreases as the strip width increases.

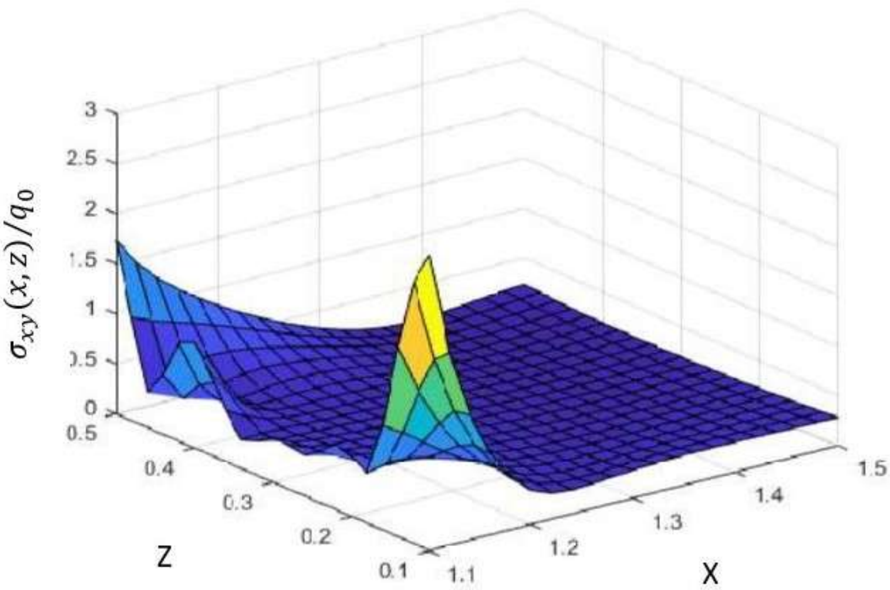

Fig.-5 Stress field outside the crack(h=1.5,k_2=0.7)

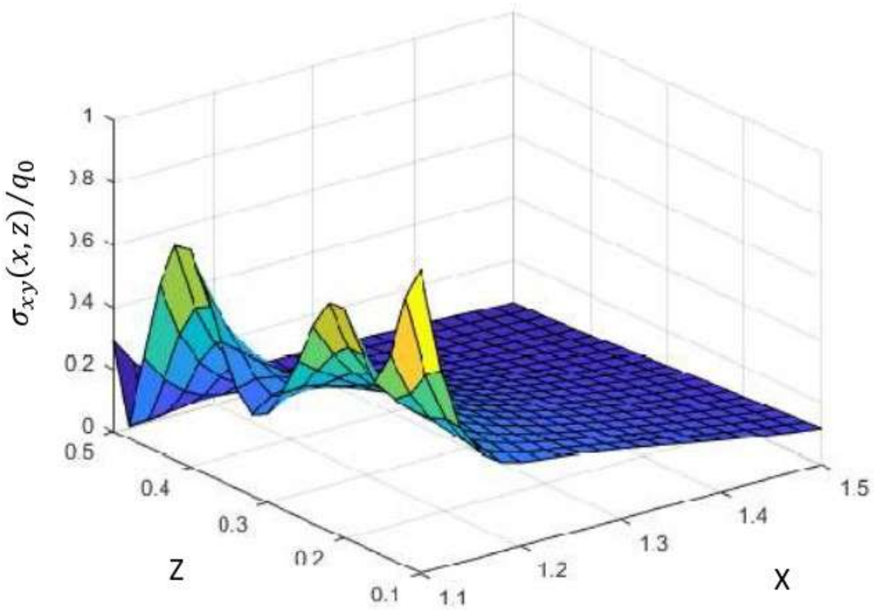

Fig.-6 Stress field outside the crack(h=3.5,k_2=0.7)

6. CONCLUSION

In this paper I have considered the problem of incident SH-wave response of a Griffith Crack in an infinitely long isotropic elastic strip. The expression of elastodynamic stress intensity factor (SIF) at the tip of the crack, shear stress away from the crack and the crack opening displacement are obtained analytically. All the quantities are evaluated numerically by solving the integral equation for different geometry parameters and shown by means of graphs. From the above study the following conclusions are drawn:

- SIF increases first with the increase in frequency then decreases.

- SIF decreases as the strip width increases.
- COD is symmetric about midpoint of the crack and reaches its maximum value here.
- COD increases for higher values of frequency upto a certain value of the frequency then starts to decrease.
- Dimensionless Stress field shows wave like nature and very disruptive near the crack tip and the sharpness decreases as x increases. It is also seen that stress decreases as the strip width increases.

This research outcome may be very significant for the study of Stress Intensity factor, crack opening displacement and stress field near the crack inthe field of fabrication processes in fracture mechanics. Further research can be doneusing different mediums, and interfaced

crack problems with the help of this work.

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